Superstatistics in high energy physics: Application to cosmic ray energy spectra and e^+e^- annihilation

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Abstract

We work out a superstatistical description of high-energy scattering processes that takes into account temperature fluctuations in small volume elements. For Γ-distributed fluctuations of the inverse temperature one effectively obtains formulas similar to those used in nonextensive statistical mechanics, whereas for other temperature distributions more general superstatistical models arise. We consider two main examples: Scattering processes of cosmic ray particles and e^+e^- annihilation processes. In both cases one obtains excellent fits of experimentally measured energy spectra and cross sections.

1 Introduction

Superstatistical techniques have been recently successfully applied to a large variety of complex systems, for example hydrodynamic turbulence [1, 2, 3, 4], defect turbulence [5], share price dynamics [6, 7], random matrix theory [8, 9], random networks [10], wind velocity fluctuations [11, 12], hydro-climatic fluctuations [13], the statistics of train departure delays [14] and models of the metastatic cascade in cancerous systems [15]. The basic idea underlying this approach is that there is an intensive parameter, for example the inverse temperature β or the energy dissipation in turbulent systems, that exhibits fluctuations on a large time scale (large as compared to internal relaxation times of the system under consideration). As a consequence, one can model these types of complex systems by a kind of superposition of ordinary statistical mechanics with varying temperature parameters, in short a superstatistics [16, 17, 18, 19, 20, 21, 22, 23]. The stationary distributions of superstatistical systems deviate from ordinary Boltzmann-Gibbs statistical mechanics and can exhibit asymptotic power laws, stretched exponentials, or other functional forms in the energy E [18].

In this paper we work out potential applications of this concept in high energy physics. In scattering processes at high energies, the effective interaction volume, as well as the number of particles involved, can be rather small, and hence temperature fluctuations can play a very important role [24, 25, 26]. These can either be temperature fluctuations from scattering event to scattering event, i.e. temporal fluctuations, or they may be related a nonequilibrium situtations where different spatial regions have different temperature, i.e. spatial fluctuations.

Our starting point for a suitable thermodynamic model will be Hagedorn's theory [27, 28], which yields a statistical description of a selfsimilar 'fireball' of particles produced in scattering events. Hagedorn's theory models the hadronization cascade from a statistical mechanics point of view (of course QCD was not known at the time when he wrote the seminal paper [27]). His theory is regularly in use to describe heavy ion collisions [29, 30, 31]. The basic assumption is that in the scattering region the density of states grows so rapidly that the effective temperature cannot exceed a certain maximum temperature, the Hagedorn temperature T_H [32]. The value is approximately $T_H \approx 180$ MeV and it describes the confinement phase transition. The Hagedorn phase transition is also of fundamental interest in string theories [33, 34, 35].

In this paper we will consider a superstatistical extension of the Hagedorn theory, starting from a q-generalized version previously introduced in [36]. The predictions of this generalized statistical mechanics model are in excellent agreement with measured experimental data. We will illustrate this for two main examples: i) observed energy spectra of cosmic rays [24] ii) experimentally measured cross sections in e^+e^- annihilation [36].

The Hagedorn theory of scattering processes is known to give correct predictions of cross sections and energy spectra for center of mass energies $E_{CMS} < 10GeV$, whereas for larger energies there is experimental evidence from various collision experiments that power-law behaviour of differential cross sections sets in. This power-law is not contained in the original Hagedorn theory but can be formally obtained if one extends the original Hagedorn theory to a superstatistical one, along the lines sketched in this paper. In fact, if fluctuations are taken into account then power laws can arise in quite a natural way in various ways, and often lead to Tsallis type of generalized distribution functions[37, 38, 39, 1, 16]. While in this paper we concentrate on cosmic ray statistics and transverse momentum spectra in e^+e^- annihilation, it is quite clear that related superstatistical techniques can be applied to other scattering data as well, for example heavy ion collisions [31, 40], pp collisions [41] and $p\bar{p}$ collisions [26].

2 Superstatistical modelling of temperature fluctuations in scattering experiments

In the superstatistics approach [16] one assumes that locally the system under consideration reaches local equilibrium but on a large spatio-temporal scale there are temperature fluctuations, described by a probability density $f(\beta)$ which describes the probability to observe a certain inverse temperature β in a given spatial area. One thus gets a kind of mixing (or superposition) of many equilibrium distributions which effectively decribe the driven nonequilibrium system with a stationary state (see [42] for a recent review discussing various applications).

For high energy scattering processes, it is clear that the larger the center of mass energy E_{CMS} of the collision process is, the smaller is the volume r^3 probed, due to the uncertainty relation $\frac{1}{c}E_{CMS} \cdot r \sim O(\hbar)$. This means, the effective interaction volume where a thermodynamic description of the

collision process makes sense will become smaller and smaller with increasing E_{CMS}. However, a smaller volume means larger temperature fluctuations. This is in particular relevant if we repeat our scattering experiment several times or if we have different scattering events in different spatial regions which all contribute to our data. It thus makes sense to consider at large energies E_{CMS} a superstatistical description of scattering events which takes into account local temperature fluctuations.

Assume that locally some value of the fluctuating inverse temperature β is given. We then expect the momentum of a randomly picked particle in this region to be distributed according to the relativistic Maxwell-Boltzmann distribution

$$p(E|\beta) = \frac{1}{Z(\beta)} E^2 e^{-\beta E}. \tag{1}$$

Here $p(E|\beta)$ denotes the conditional probability to observe a particle with energy $E = \sqrt{\vec{p}^2 c^2 + m^2 c^4}$, given some value of β. For highly relativistic particles we can neglect the rest mass m so that $E = c|\vec{p}|$, where $\vec{p}$ is the momentum. The normalization constant is given by

$$Z(\beta) = \int_0^\infty E^2 e^{-\beta E} dE = \frac{2}{\beta^3}. \tag{2}$$

Now let us take into account local temperature fluctuations in the small interaction volumes where scattered particles are produced. We have to consider some suitable probability density $f(\beta)$ of the inverse temperature in the various interaction volumes. In a long series of experiments we will then observe the marginal distribution obtained by integrating over all β

$$p(E) = \int_0^\infty p(E|\beta) f(\beta) d\beta. \tag{3}$$

These types of distributions are generally studied in superstatistical models, and are known to exhibit fat tails whose asymptotic decay with E depends on how $f(\beta)$ behaves for $\beta \to 0$ (more details on this in [18]). Of course, the validity of the superstatistical modelling approach requires sufficient time scale separation so that the system relaxes to local equilibrium fast enough before the next temperature fluctuation takes place [3, 7, 20, 23].

It has been argued in previous work [3] that there are basically three different distributions $f(\beta)$ that are relevant for large classes of complex systems: These are the χ^2-distribution, the inverse χ^2 distribution, and the

lognormal distribution. Lognormal superstatistics is often observed in hydrodynamic turbulence, due to the multiplicative nature of the Richardson cascade [3, 4]. Inverse χ^2 superstatistics plays an important role in random matrix theory [8, 9], as well as in medical statistics [15]. For high energy physics, χ^2 superstatistics is most relevant, although other superstatistics may play a role as well (for example, one could think about a superstatistics generating Kaniadakis statistics [43]).

For χ^2 superstatistics (or equivalently Γ superstatistics), the distribution $f(\beta)$ is given by the χ^2-distribution of degree n, i.e.

$$f(\beta) = \frac{1}{\Gamma\left(\frac{n}{2}\right)} \left\{ \frac{n}{2\beta_0} \right\}^{\frac{n}{2}} \beta^{\frac{n}{2}-1} \exp\left\{ -\frac{n\beta}{2\beta_0} \right\}. \tag{4}$$

The χ^2-distribution is a typical distribution that naturally arises in many circumstances, for example if n independent Gaussian random variables $X_i,\ i = 1, \ldots, n$ with average 0 and the same variance are squared and added. If we write

$$\beta := \sum_{i=1}^{n} X_i^2 \tag{5}$$

then β has the probability density function (4). The average of the fluctuating β is given by

$$\langle \beta \rangle = n \langle X_i^2 \rangle = \int_0^\infty \beta f(\beta) d\beta = \beta_0 \tag{6}$$

and the variance by

$$\langle \beta^2 \rangle - \beta_0^2 = \frac{2}{n} \beta_0^2. \tag{7}$$

The integral (3) with $f(\beta)$ given by (4) and $p(E|\beta)$ given by (1) is easily evaluated and one obtains

$$p(E) \sim \frac{E^2}{(1 + b(q-1)E)^{\frac{1}{q-1}}} \tag{8}$$

where

$$q = 1 + \frac{2}{n+6} \tag{9}$$

and

$$b = \frac{\beta_0}{4 - 3q}. \tag{10}$$

Note that the partition function $Z(\beta)$ entering into eq. (3) is β-dependent. Different β-dependencies of the partition function $Z(\beta)$ lead to different answers if the integration over β is performed. In particular, the precise relation between q and n depends on this. This was for the first time correctly worked out in [1].

The distribution (8) is a q-generalized relativistic Maxwell-Boltzmann distribution in the formalism of nonextensive statistical mechanics [37]. These kind of distributions can be directly obtained by maximizing the q-entropies [37]

$$S_q = k\frac{1}{q-1}(1-\sum_i p_i^q) \tag{11}$$

and multiplying with the available phase space volume. The p_i are the probabilities of the microstates i. The q-entropies contain the Shannon entropy $S_1 = -k\sum_i p_i \ln p_i$ underlying ordinary statistical mechanics as a special case for $q = 1$. Whereas $q = 1$ corresponds to the usual canonical ensemble with constant temperature, the Tsallis-canonical ensemble obtained for $q > 1$ is capable of describing temperature fluctuations, assuming that β is χ^2-distributed. We thus have a plausible physical mechanism why a Tsallis-like statistical description makes sense if a suitable intensive parameter fluctuates [1, 38]. Tsallis statistics with $q = 3$ (respectively $q = -1$ if the escort formalism [44, 45] is used) also plays an important role for chaotically quantized scalar fields [46, 47], dark energy models [48] and moduli field dynamics [49].

3 Superstatistical partition functions

When experimental scattering data are collected, one often looks at momentum distributions of a particular particle that are produced by repeating the same experiment many times. In each scattering event, the effective temperature (given by the heat bath of surrounding particles) will fluctuate from event to event. Since the scattering experiment is repeated many times in an independent way, and since we concentrate on the statistics of just one particle rather then many-particle states, it makes sense to directly integrate out the temperature fluctuations and to consider effective 1-particle superstatistical Boltzmann factors.

Let us consider particles of different types and label the particle types by an index j. Each particle can be in a certain momentum state labelled by

the index i. The energy associated with this state is

$$\epsilon_{ij} = \sqrt{\vec{p}_i^2 + m_j^2}, \tag{12}$$

using units where $c = 1$. We may now define an effective 1-particle Boltzmann factor by[1]

$$x_{ij} := \int_0^\infty d\beta \, f(\beta) e^{-\beta \epsilon_{ij}}. \tag{13}$$

For example, for Tsallis statistics $f(\beta)$ is a χ^2 distribution and one has

$$x_{ij} = (1 + (q-1) b \epsilon_{ij})^{-\frac{1}{q-1}}, \tag{14}$$

where b^{-1} is proportional to the average temperature, and $q - 1$ is a measure of the strength of inverse temperature fluctuations. The effective Boltzmann factor x_{ij} approaches the ordinary Boltzmann factor $e^{-b\epsilon_{ij}}$ for $q \to 1$.

The simplest way to introduce a generalized grand canonical partition function (which, as said before, regards all temperature fluctuations to be independent) is as follows:

$$Z = \sum_{(\nu)} \prod_{ij} x_{ij}^{\nu_{ij}} \tag{15}$$

Here ν_{ij} denotes the number of particles of type j in momentum state i. The sum $\sum_{(\nu)}$ stands for a summation over all possible particle numbers. For bosons one has $\nu_{ij} = 0, 1, 2, \ldots, \infty$, whereas for fermions one has $\nu_{ij} = 0, 1$. It follows that for bosons

$$\sum_{\nu_{ij}} x_{ij}^{\nu_{ij}} = \frac{1}{1 - x_{ij}} \qquad (bosons) \tag{16}$$

whereas for fermions

$$\sum_{\nu_{ij}} x_{ij}^{\nu_{ij}} = 1 + x_{ij} \qquad (fermions) \tag{17}$$

Hence the partition function can be written as

$$Z = \prod_{ij} \frac{1}{1 - x_{ij}} \prod_{i'j'} (1 + x_{i'j'}), \tag{18}$$

[1] To describe 2-particle states with the same temperature fluctuations surrounding both particles one has to proceed in a slightly different way, see [1].

where the prime labels fermionic particles. For the logarithm of the partition function we obtain

$$\log Z = -\sum_{ij} \log(1 - x_{ij}) + \sum_{i'j'} \log(1 + x_{i'j'}). \tag{19}$$

One may actually proceed to continuous variables by replacing

$$\sum_i [...] \to \int_0^\infty \frac{V_0 4\pi p^2}{h^3} [...] dp = \frac{V_0}{2\pi^2} \int_0^\infty p^2 [...] dp \quad (\hbar = 1) \tag{20}$$

(V_0: volume of the interaction region) and

$$\sum_j [...] \to \int_0^\infty \rho(m) [...] dm, \tag{21}$$

where $\rho(m)$ is the mass spectrum.

Let us now formally calculate the average occupation number of a particle of species j in the momentum state i. We obtain

$$\bar{\nu_{ij}} = x_{ij} \frac{\partial}{\partial x_{ij}} \log Z = \frac{x_{ij}}{1 \pm x_{ij}}. \tag{22}$$

For example, for the case of Tsallis statistics one obtains

$$\bar{\nu_{ij}} = \frac{1}{(1 + (q-1) b \epsilon_{ij})^{\frac{1}{q-1}} \pm 1} \tag{23}$$

where the $-$ sign is for bosons and the $+$ sign for fermions.

In order to single out a particular particle of mass m_0, one can formally work with the mass spectrum $\rho(m) = \delta(m - m_0)$. To obtain the probability to observe a particle of mass m_0 in a certain momentum state, we have to multiply the average occupation number with the available volume in momentum space. An infinitesimal volume in momentum space can be written as

$$dp_x dp_y dp_z = dp_L p_T \sin\theta dp_T d\theta \tag{24}$$

where $p_T = \sqrt{p_y^2 + p_z^2}$ is the transverse monentum and $p_x = p_L$ is the longitutinal one. By integrating over all θ and p_L one finally arrives at a probability density $w(p_T)$ of transverse momenta given by

$$w(p_T) = const \cdot p_T \int_0^\infty dp_L \frac{1}{(1 + (q-1) b \sqrt{p_T^2 + p_L^2 + m_0^2})^{\frac{1}{q-1}} \pm 1}. \tag{25}$$

Since the Hagedorn temperature is rather small (of the order of the π mass), and b^{-1} is of the order of the Hagedorn temperature, under normal circumstances one has $b\sqrt{p_T^2+p_L^2+m_0^2} >> 1$, and hence the ± 1 can be neglected if q is close to 1. One thus obtains for both fermions and bosons the statistics

$$w(p_T) \approx const \cdot p_T \int_0^\infty dp_L \left(1+(q-1)b\sqrt{p_T^2+p_L^2+m_0^2}\right)^{-\frac{1}{q-1}} \tag{26}$$

which, if our model assumptions are satisfied, should determine the p_T dependence of experimentally measured particle spectra. The differential cross section $\sigma^{-1}d\sigma/dp_T$ is expected to be proportional to $w(p_T)$.

In nonextensive statistical mechanics, it is sometimes of advantage to consider normalized q-expectation values [45], thus implementing a formalism that is based on so-called escort distributions [44]. If the escort formalism is used then the power $\frac{1}{q-1}$ in the above formula is replaced by $\frac{q}{q-1}$, a simple reparametrization. For more general classes of superstatistics described by general $f(\beta)$ the q-exponential is replaced by $\int_0^\infty f(\beta)\exp\{-\beta\sqrt{p_T^2+p_L^2+m_0^2}\}d\beta$.

4 Comparison with measured cosmic ray energy spectra

Let us now look at measured data. Fig. 1 shows the experimentally measured energy spectrum of primary cosmic ray particles as observed on the earth [50, 51, 52, 53, 54, 55]. The measured spectrum is very well fitted over many decades of different energies by the distribution (8) [24] or other generalized nonextensive distributions [56]. The best fit is obtained if the entropic index q is chosen as

$$q = 1.215 \tag{27}$$

and if the effective temperature parameter is given by

$$kT_0 = b^{-1} = 107 \text{ MeV}. \tag{28}$$

Let us now predict a plausible value of q using the superstatistical approach. The variables X_i in eq. (5) describe the independent degrees of freedom contributing to the fluctuating inverse temperature β. At very large center of mass energies $E_{CMS} \to \infty$, the interaction region is very small, and all relevant degrees of freedom are basically represented by the 3 spatial

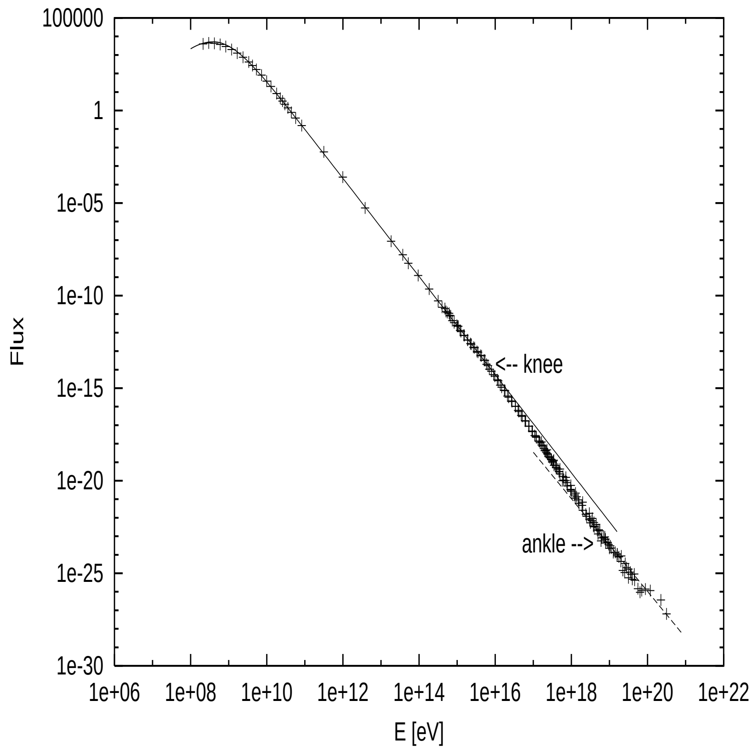

Figure 1: Measured energy spectrum of primary cosmic rays (in units of $m^{-3}s^{-1}sr^{-1}GeV^{-1}$) [24]. The solid line is the formula (8) with $q = 1.215$, $b^{-1} = kT_0 = 107$ MeV and $C = 5 \cdot 10^{-13}$ in the above units. The dashed line is eq. (8) with $q = 11/9$, $kT_0 = 107$ MeV and C smaller by a factor 1/50.

dimensions into which heat can flow. We may physically interpret X_i^2 as the heat loss in the spatial i-direction, $i = x, y, z$, during the collision process that generates the primary cosmic ray particle. The more heat is lost, the smaller is the local kT, i.e. the larger is the local β given by (5). The 3 spatial degrees of freedom yield $n = 3$ as the smallest possible value of n or, according to (9),

$$q = \frac{11}{9} = 1.222. \tag{29}$$

For cosmic rays E_{CMS} is very large, hence we expect a q-value that is close to this asymptotic value. The fit in Fig. 1 in fact uses $q = 1.215$, which agrees with the predicted value in eq. (29) to about 3 digits (similar q-values were also obtained in [56]).

For smaller center of mass energies, according to $\frac{1}{c}E_{CMS} \cdot r \sim O(\hbar)$, the interaction region will be bigger and more effective degrees of freedom within this bigger interaction region will contribute to the fluctuating temperature. Hence we expect that for smaller E_{CMS} n will be larger than 3, or q will be smaller than 11/9.

Finally, for $E_{CMS} \to 0$, $q \to 1$ and ordinary statistical mechanics is recovered. In this classical limit case, the relevant interaction region r^3 where a thermodynamic description makes sense becomes very large, and within this large region a large number of independent degrees of freedom n contributes to the fluctuating temperature, represented by many different particles. According to eq. (9), the limit $n \to \infty$ is equivalent to $q \to 1$ and in this limit the χ^2-distribution degenerates to a delta function $\delta(\beta - \beta_0)$.

It is reasonable to assume [51] that the 'knee' at $E \approx 10^{16}$ eV is due to the fact that one has reached the maximum energy scale to which typical galactic accelerators can accelerate. This then implies a rapid fall in the number of observed events with a higher energy, i.e. a steeper slope in Fig. 1 between about 10^{16} and 10^{19} eV. The 'ankle' at $E \approx 10^{19}$ eV may then be due to the fact that a higher energy population of cosmic ray particles takes over from a lower energy population. This higher energy population may have a different origin (for example, extragalactic origin). The new population has a significantly smaller flux rate but can reach much larger energies. As a matter of fact the cosmic accelerators underlying the production process of this new species of cosmic rays must have a much larger center of mass energy E_{CMS} than the ankle energy $\sim 10^{19}$ eV, so q should be given by its asymptotic value 11/9, whereas the effective temperature T_0 should be the same as before. The dashed line in Fig. 1 corresponds to our formula with

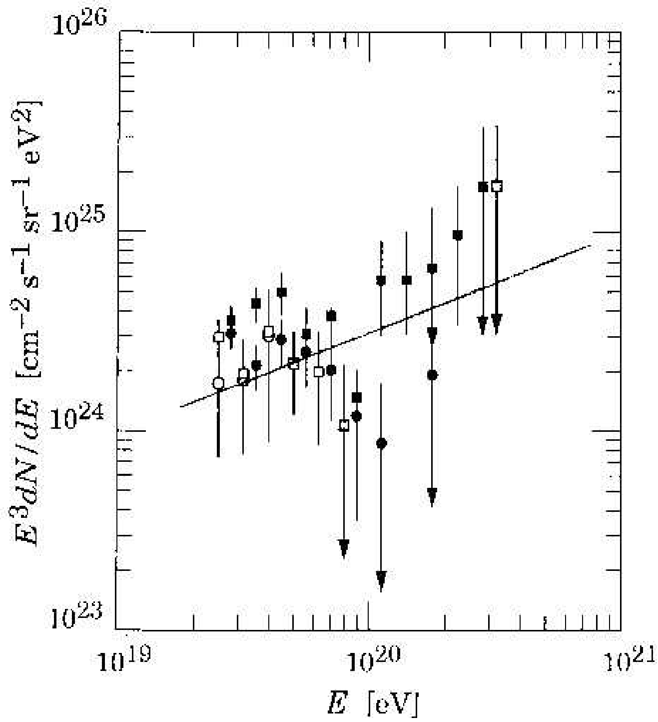

Figure 2: Measured cosmic ray energy spectrum $E^3 \cdot dN/dE$ at largest energies (data from [51, 52, 53, 54]). The straight line is a power law with exponent $\alpha = 5/2$ (corresponding to $q = 11/9$).

$q = 11/9$, $k\tilde{T} = 107$ MeV and a flux rate that is smaller by a factor $1/50$ as compared to the high-flux generation of cosmic rays. This is consistent with the data.

Formula (8) predicts asymptotic power-law behavior of the measured energy spectrum. For large E one has $p(E) \sim E^{-\alpha}$ where the index α is given by

$$\alpha = \frac{1}{q-1} - 2. \tag{30}$$

$q = 1.215$ implies $\alpha = 2.65$ (for moderately large energies), whereas for largest energies the asymptotic value $q = 11/9$ implies $\alpha = 5/2$. As shown in Fig. 2, the largest-energy events are compatible with such an asymptotic power law exponent.

5 Comparison with experimentally measured differential cross sections in e^+e^- annihilation

Let us now proceed to our second example, differential cross sections for transverse momenta in e^+e^- annihilation experiments [36, 57]. If one uses the escort formalism [44, 45] then formula (26) implies

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T} = Cu\int_0^\infty dx\ \left(1+(q-1)\sqrt{x^2+u^2+m_\beta^2}\right)^{-\frac{q}{q-1}} \tag{31}$$

Here $x = p_L/T_0$, $u = p_T/T_0$ and $m_\beta := m_0/T_0$ are the longitudinal momentum, transverse momentum and mass in units of a temperature parameter T_0 that is of the same order of magnitude as the Hagedorn temperature. C is a suitable constant related to multiplicity.

Generally, one knows that the interaction energy in a nonextensive system increases with increasing entropic index q. Since this energy must be taken from somewhere, it is most natural to assume that it is taken from the heat bath. So the average temperature parameter T_0 should slighly decrease with increasing q. In [36] a linear dependence of T_0 on q was postulated, of the simple form

$$T_0 = \left(1-\frac{q}{3}\right)T_H \tag{32}$$

where $T_H = 180\ MeV$ is the Hagedorn temperature.

We also have to clarify the dependence of q on E_{CMS}. Clearly, for $E_{CMS}\to\infty$ our argument in section 2 and 4 suggests the value $q = 11/9$, whereas for $E_{CMS}\to 0$ one has $q = 1$, i.e the ordinary Hagedorn theory without fluctuations. In [36], a smooth interpolation between these values was suggested, of the form

$$q(E_{CMS}) = \frac{11-e^{-E_{CMS}/E_0}}{9+e^{-E_{CMS}/E_0}} \tag{33}$$

where $E_0 \approx 45.6\,GeV$ is about half of the Z_0 mass.

Finally, one also needs to know the multiplicity (the average number of produced charged particles) in order to estimate the cross section. The multiplicity M as a function of E_{CMS} has been independently measured in many experiments[58]. A good fit of the experimental data in the relevant

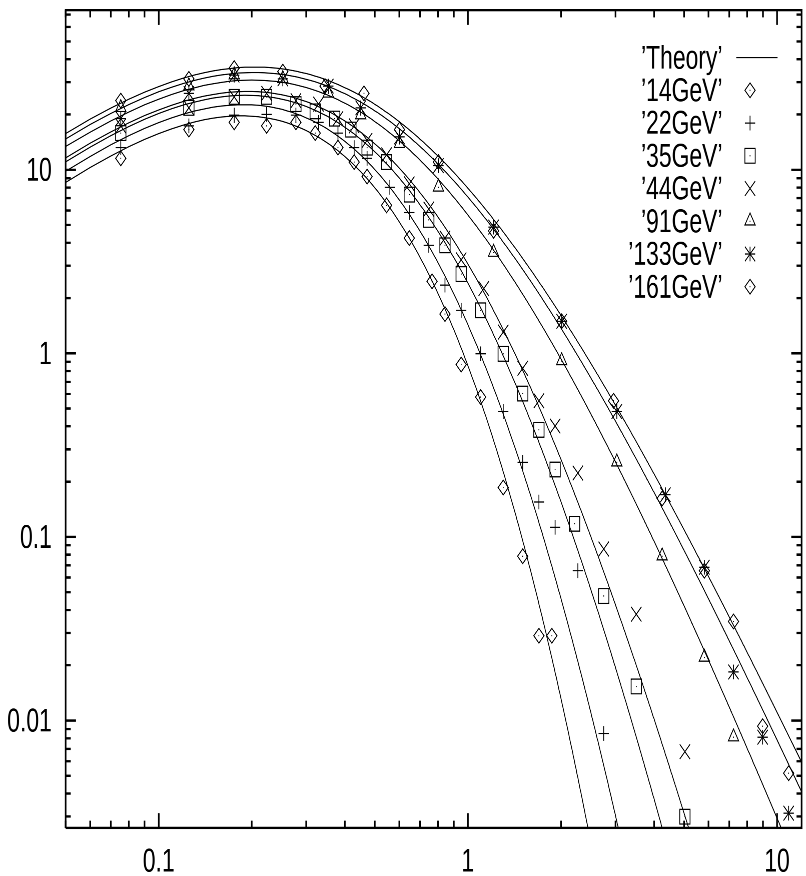

Figure 3: Differential cross section as a function of the transverse momentum p_T for various center of mass energies E. The data correspond to measurements of the TASSO ($E \leq 44$ GeV) and DELPHI ($E \geq 91$ GeV) collaboration. The solid lines are given by the analytic formula (35).

energy region is the formula [36]

$$M = \left(\frac{E_{CMS}}{T_0^{q=1}} \right)^{5/11} \tag{34}$$

The final formula derived for the cross section in [36] involves a further approximation step to perform the integration over x and is then finally given by

$$\frac{1}{\sigma} \frac{d\sigma}{dp_T} = \frac{1}{T_0} M p(u). \tag{35}$$

where $p(u)$ is the normalized probability density

$$p(u) = \frac{1}{Z_q} \, u^{3/2} \left(1 + (q-1)u\right)^{-\frac{q}{q-1}+\frac{1}{2}} \tag{36}$$

with normalization constant

$$Z_q = (q-1)^{-5/2} B\left(\frac{5}{2}, \frac{q}{q-1} - 3 \right). \tag{37}$$

Formula (35) with $p(u)$ given by eq. (36), $q(E_{CMS})$ given by eq. (33), $T_0(q)$ given by eq.(32) and multiplicity $M(E_{CMS})$ given by eq. (34) turns out to very well reproduce the experimental results of measured transversal cross sections for all energies E_{CMS}. This is illustrated in Fig. 3 which shows the measured differential cross section versus p_T. The solid lines are given by formula (35) for the various center of mass energies E_{CMS}. One obtains excellent agreement with the measured data of the TASSO and DELPHI collaborations [59, 60]. Remarkably, for the largest energies the best fitting parameter q is again given by $q \approx 11/9$, in agreement with the data for the cosmic rays. Moreover, at the largest energies, according to eq. (32) T_0 is given by 107 MeV, again in agreement with what was used for the cosmic ray data in Fig. 1. Our superstatistical argument in section 2 and 4, relating q to temperature fluctuatuons and predicting from this $q \approx 11/9$ for largest energies E_{CMS}, is indeed generally applicable.

6 Conclusion

In this paper we have dealt with a superstatistical generalization of the Hagedorn theory which is generally applicable to describe the statistics of scattered

particles produced in high-energy collisions. Our approach is based on taking into account temperature fluctuations in small interaction volumes. For χ^2-distributed inverse temperature one effectively ends up with formulas that are similar to those used in non-extensive versions of statistical mechanics.

At large energies the χ^2-superstatistical approach implies that energy spectra of particles and cross sections decay with a power law. This power law is indeed observed for various experimental data. We obtained formulas that are in very good agreement with experimentally measured data for cosmic rays and e^+e^- annihilation. In particular, the superstatistical approach allowed us to give a concrete prediction for q at largest center of mass energies, namely $q \approx 11/9$. Generally, it appears that high-energy scattering data do not only yield valuable information on elementary particle physics, but they may also be regarded as test grounds to further develop generalized versions of statistical mechanics.

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